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Chapter 6

LEAKAGE INDUCTANCES AND RESISTANCES

6.1. LEAKAGE FIELDS

Any magnetic field (H_i , B_i) zone within the IM is characterized by its stored magnetic energy (or coenergy) W_m .

$$W_{mi} = \frac{1}{2} \iiint_V (\vec{B} \cdot \vec{H}) dV = L_i \frac{I_i^2}{2} \quad (6.1)$$

Equation (6.1) is valid when, in that region, the magnetic field is produced by a single current source, so an inductance “translates” the field effects into circuit elements.

Besides the magnetic energy related to the magnetization field (investigated in Chapter 5), there are flux lines that encircle only the stator or only the rotor coils (Figure 6.1). They are characterized by some equivalent inductances called leakage inductances L_{sl} , L_{rl} .

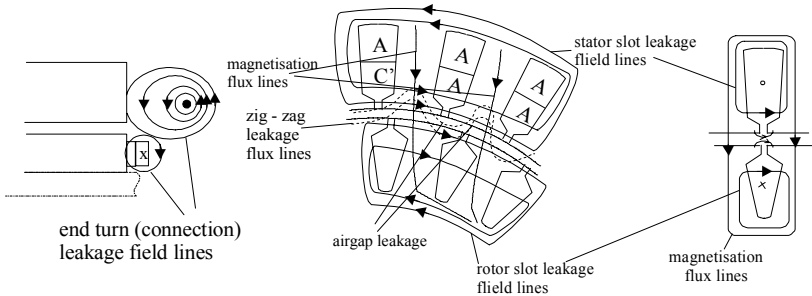


Figure 6.1 Leakage flux lines and components

There are leakage flux lines which cross the stator and, respectively, the rotor slots, end-turn flux lines, zig-zag flux lines, and airgap flux lines (Figure 6.1). In many cases, the differential leakage is included in the zig-zag leakage. Finally, the airgap flux space harmonics produce a stator emf as shown in Chapter 5, at power source frequency, so it should also be considered in the leakage category. Its torques occur at low speeds (high slips) and thus are not there at no-load operation.

6.2. DIFFERENTIAL LEAKAGE INDUCTANCES

As both the stator and rotor currents may produce space flux density harmonics in the airgap (only step mmf harmonics are considered here), there will be both a stator and a rotor differential inductance. For the stator, it is sufficient to add all L_{1mv} harmonics, but the fundamental (5.122), to get L_{ds} .

$$L_{ds} = \frac{6\mu_0\tau L_c W_l^2}{\pi^2 p_1 g K_c} \sum_{v \neq 1} \frac{K_{wv}^2}{v^2 K_{sv}}; \quad v = Km_1 \pm 1 \quad (6.2)$$

The ratio σ_d of L_{ds} to the magnetization inductance L_{1m} is

$$\sigma_{ds0} = \frac{L_{ds}}{L_{1m}} = \sum_{v \neq 1} \left(\frac{K_{wsv}^2}{v^2 K_{wsl}^2} \right) \cdot \frac{K_s}{K_{sv}} \quad (6.3)$$

where K_{wsv} , K_{wsl} are the stator winding factors for the harmonic v and for the fundamental, respectively.

K_s and K_{sv} are the saturation factors for the fundamental and for harmonics, respectively.

As the pole pitch of the harmonics is τ/v , their fields do not reach the back cores and thus their saturation factor K_{sv} is smaller than K_s . The higher v , the closer K_{sv} is to unity. In a first approximation,

$$K_{sv} \approx K_{st} < K_s \quad (6.4)$$

That is, the harmonics field is retained within the slot zones so the teeth saturation factor K_{st} may be used (K_s and K_{st} have been calculated in Chapter 5). A similar formula for the differential leakage factor can be defined for the rotor winding.

$$\sigma_{dr0} = \sum_{\mu \neq 1} \left(\frac{K_{wr\mu}^2}{\mu^2 K_{wrl}^2} \right) \cdot \frac{K_s}{K_{s\mu}} \quad (6.5)$$

As for the stator, the order μ of rotor harmonics is

$$\mu = K_2 m_2 \pm 1 \quad (6.6)$$

m_2 —number of rotor phases; for a cage rotor $m_2 = Z_2/p_1$, also $K_{wrl} = K_{wr\mu} = 1$.

The infinite sums in (6.3) and (6.5) are not easy to handle. To avoid this, the airgap magnetic energy for these harmonics fields can be calculated. Using (6.1)

$$B_{gv} = \frac{\mu_0 F_{mv}}{g K_c K_{sv}} \quad (6.7)$$

We consider the step-wise distribution of mmf for maximum phase A current, (Figure 6.2), and thus

$$\sigma_{ds0} = \frac{\int_0^{2\pi} F^2(\theta) d\theta}{F_{lm}^2 2\pi} = \frac{1}{N_s} \frac{\sum_{j=1}^{N_s} F_j^2(\theta)}{F_{lm}^2} \quad (6.8)$$

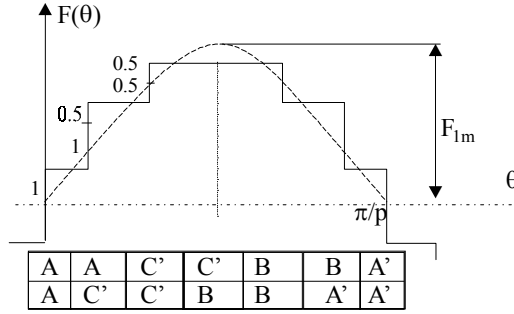


Figure 6.2 Step-wise mmf waveform ($q = 2$, $y/\tau = 5/6$)

The final result for the case in Figure 6.2 is $\sigma_{ds0} = 0.0285$.

This method may be used for any kind of winding once we know the number of turns per coil and its current in every slot.

For full-pitch coil three-phase windings [1],

$$\sigma_{ds} \approx \frac{5q^2 + 1}{12q^2} \cdot \frac{2\pi^2}{m_1^2 K_{w1}^2} - 1 \quad (6.9)$$

Also, for standard two-layer windings with chorded coils with chording length ε , σ_{ds} is [1]

$$\sigma_{ds} = \frac{2\pi^2}{m_1^2 K_{w1}^2} \cdot \frac{5q^2 + 1 - \frac{3}{4} \left(1 - \frac{y}{\tau}\right) \left[9q^2 \left(1 - \left(\frac{y}{\tau}\right)^2\right) + 1 \right]}{12q^2} - 1 \quad (6.10)$$

In a similar way, for the cage rotor with skewed slots,

$$\sigma_{r0} = \frac{1}{K_{skew}'^2 \eta_{r1}^2} - 1 \quad (6.11)$$

with

$$K_{skew}' = \frac{\sin\left(\alpha_{er} \cdot \frac{c}{\tau_r}\right)}{\alpha_{er} \cdot \frac{c}{\tau_r}}; \quad \eta_{r1} = \frac{\sin(\alpha_{er}/2)}{\alpha_{er}/2}; \quad \alpha_{er} = 2\pi \frac{p_l}{N_r} \quad (6.12)$$

The above expressions are valid for three-phase windings. For a single-phase winding, there are two distinct situations. At standstill, the a.c. field produced by one phase is decomposed into two equal traveling waves. They both produce a differential inductance and, thus, the total differential leakage inductance $(L_{ds1})_{s=1} = 2L_{ds}$.

On the other hand, at $S = 0$ (synchronism) basically the inverse (backward) field wave is almost zero and thus $(L_{ds1})_{S=0} \approx L_{ds}$.

The values of differential leakage factor σ_{ds} (for three- and two-phase machines) and σ_{dr} , as calculated from (6.9) and (6.10) are shown on [Figures 6.3](#) and [6.4](#). [1]

A few remarks are in order.

- For $q = 1$, the differential leakage coefficient σ_{ds0} is about 10%, which means it is too large to be practical.
- The minimum value of σ_{ds0} is obtained for chorded coils with $y/\tau \approx 0.8$ for all q_s (slots/pole/phase).
- For same q , the differential leakage coefficient for two-phase windings is larger than for three-phase windings.
- Increasing the number of rotor slots is beneficial as it reduces σ_{dr0} ([Figure 6.4](#)).

[Figures 6.3](#) do not contain the influence of magnetic saturation. In heavily saturated teeth IMs as evident in (6.3), $K_s/K_{st} > 1$, the value of σ_{ds} increases further.

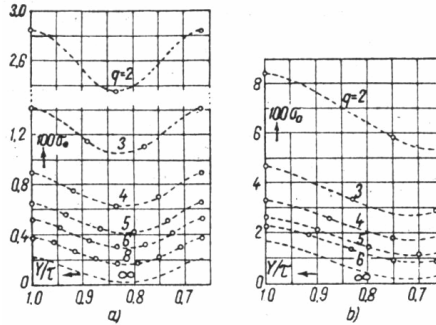


Figure 6.3 Stator differential leakage coefficient σ_{ds} for three phases a.) and two-phases b.) for various q_s

The stator differential leakage flux (inductance) is attenuated by the reaction of the rotor cage currents.

Coefficient Δ_d for the stator differential leakage is [1]

$$\Delta_d \approx 1 - \frac{1}{\sigma_{ds0}} \sum_{v \neq 1}^{2mq_1+1} \left(K'_{skewv} \eta_{rv} \frac{K_{wv}}{vK_{w1}} \right)^2; \quad \sigma_{ds} = \sigma_{ds0} \Delta_d \quad (6.13)$$

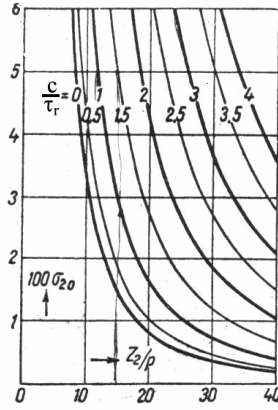


Figure 6.4 Rotor cage differential leakage coefficient σ_{dr} for various q_s and straight and single slot pitch skewing rotor slots ($c/\tau_r = 0, 1, \dots$)

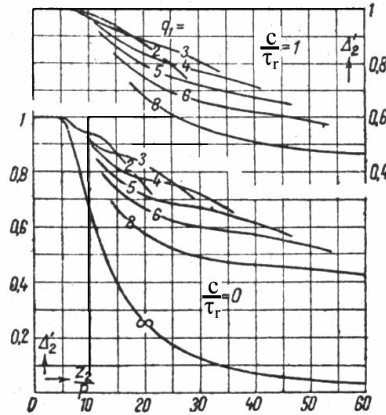


Figure 6.5. Differential leakage attenuation coefficient Δ_d for cage rotors with straight ($c/\tau_r = 0$) and skewed slots ($c/\tau_r = 1$)

As the stator winding induced harmonic currents do not attenuate the rotor differential leakage: $\sigma_{dr} = \sigma_{dr0}$.

A rather complete study of various factors influencing the differential leakage may be found in [Reference 2].

Example 6.1. For the IM in Example 5.1, with $q = 3$, $N_s = 36$, $2p_1 = 4$, $y/\tau = 8/9$, $K_{w1} = 0.965$, $K_s = 2.6$, $K_{st} = 1.8$, $N_r = 30$, stack length $L_e = 0.12\text{m}$, $L_{1m} = 0.1711\text{H}$, $W_1 = 300$ turns/phase, let us calculate the stator differential leakage inductance L_{ds} including the saturation and the attenuation coefficient Δ_d of rotor cage currents.

Solution

We will find first from Figure 6.3 (for $q = 3$, $y/\tau = 0.88$) that $\sigma_{ds0} = 1.16 \cdot 10^{-2}$. Also from Figure 6.5 for $c = 1\tau_s$ (skewing), $Z_2/p_1 = 30/2 = 15$, $\Delta_d = 0.92$. Accounting for both saturation and attenuation coefficient Δ_d , the differential leakage stator coefficient K_{ds} is

$$K_{ds} = K_{ds0} \cdot \frac{K_s}{K_{st}} \Delta_d = 1.16 \cdot 10^{-2} \cdot \frac{2.6}{1.8} \cdot 0.92 = 1.5415 \cdot 10^{-2} \quad (6.14)$$

Now the differential leakage inductance L_{ds} is

$$L_{ds} = K_{ds} L_{1m} = 1.5415 \cdot 10^{-2} \cdot 0.1711 = 0.2637 \cdot 10^{-3} \text{ H} \quad (6.15)$$

As seen from (6.13), due to the rather large q , the value of K_{ds} is rather small, but, as the number of rotor slots/pole pair is small, the attenuation factor Δ_d is large.

Values of $q = 1, 2$ lead to large differential leakage inductances. The rotor cage differential leakage inductance (as reduced to the stator) L_{dr} is

$$L_{dr} = \sigma_{dr0} \frac{K_s}{K_{ts}} L_{1m} = 2.8 \cdot 10^{-2} \cdot \frac{2.6}{1.8} L_{1m} = 4.04 \cdot 10^{-2} L_{1m} \quad (6.16)$$

σ_{dr0} is taken from Figure 6.4 for $Z_2/p_1 = 15$, $c/\tau_r = 1$, : $\sigma_{dr0} = 2.8 \cdot 10^{-2}$.

It is now evident that the rotor (reduced to stator) differential leakage inductance is, for this case, notable and greater than that of the stator.

6.3. RECTANGULAR SLOT LEAKAGE INDUCTANCE/SINGLE LAYER

The slot leakage flux distribution depends notably on slot geometry and less on teeth and back core saturation. It also depends on the current density distribution in the slot which may become nonuniform due to eddy currents (skin effect) induced in the conductors in slot by their a.c. leakage flux.

Let us consider the case of a rectangular stator slot where both saturation and skin effect are neglected (Figure 6.6).

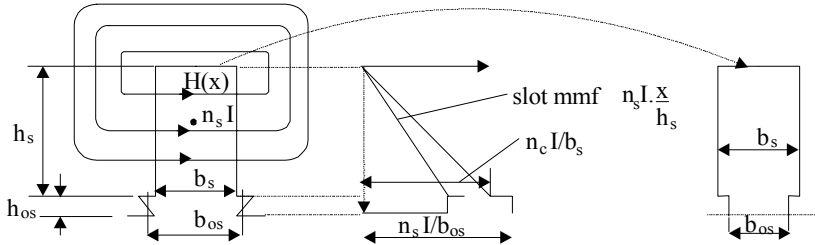


Figure 6.6 Rectangular slot leakage

Ampere's law on the contours in [Figure 6.6](#) yields

$$\begin{aligned} H(x)b_s &= \frac{n_s \cdot i \cdot x}{h_s} ; 0 \leq x \leq h_s \\ H(x)b_s &= n_s \cdot i ; h_s \leq x \leq h_s + h_{os} \end{aligned} \quad (6.17)$$

The leakage inductance per slot, L_{sls} , is obtained from the magnetic energy formula per slot volume.

$$L_{sls} = \frac{2}{i^2} W_{ms} = \frac{2}{i^2} \cdot \frac{1}{2} \int_0^{h_s+h_{os}} \mu_0 H(x)^2 dx \cdot L_e b_s = \mu_0 n_s^2 L_e \left[\frac{h_s}{3b_s} + \frac{h_{os}}{b_{os}} \right] \quad (6.18)$$

The term in square parenthesis is called the geometrical specific slot permeance.

$$\lambda_s = \frac{h_s}{3b_s} + \frac{h_{os}}{b_{os}} \approx 0.5 \div 2.5; h_{os} = (1 \div 3) 10^{-3} \text{ m} \quad (6.19)$$

It depends solely on the aspect of the slot. In general, the ratio $h_s/b_s < (5-6)$ to limit the slot leakage inductance to reasonable values.

The machine has N_s stator slots and N_s/m_1 of them belong to one phase. So the slot leakage inductance per phase L_{sl} is

$$L_{sl} = \frac{N_s}{m_1} L_{sls} = \frac{2p_1 q m_1}{m_1} L_{sls} = 2\mu_0 W_1^2 L_e \frac{\lambda_s}{p_1 q} \quad (6.20)$$

The wedge location has been replaced by a rectangular equivalent area on [Figure 6.6](#). A more exact approach is also possible.

The ratio of slot leakage inductance L_{sl} to magnetizing inductance L_{lm} is (same number of turns/phase),

$$\frac{L_{sl}}{L_{lm}} = \frac{\pi^2}{3} \frac{g K_c K_s}{(K_{w1})^2} \frac{\tau}{q} \lambda_s \quad (6.21)$$

Suppose we keep a constant stator bore diameter D_i and increase two times the number of poles.

The pole pitch is thus reduced two times as $\tau = \pi D / 2p_1$. If we keep the number of slots constant q will be reduced twice and, if the airgap and the winding factor are the same, the saturation stays low for the low number of poles. Consequently, L_{sl}/L_{lm} increases two times (as λ_s is doubled for same slot height).

Increasing q (and the number of slots/pole) is bound to reduce the slot leakage inductance (6.20) to the extent that λ_s does not increase by the same ratio. Our case here refers to a single-layer winding and rectangular slot.

Two-layer windings with chorded coils may be investigated the same way.

6.4. RECTANGULAR SLOT LEAKAGE INDUCTANCE/TWO LAYERS

We consider the coils are chorded (Figure 6.7).

Let us consider that both layers contribute a field in the slot and add the effects. The total magnetic energy in the slot volume is used to calculate the leakage inductance L_{sls} .

$$L_{sls} = \frac{2L_e}{i^2} \int_0^{h_{st}} \mu_0 [H_1(x) + H_2(x)]^2 dx \cdot b(x) \quad (6.22)$$

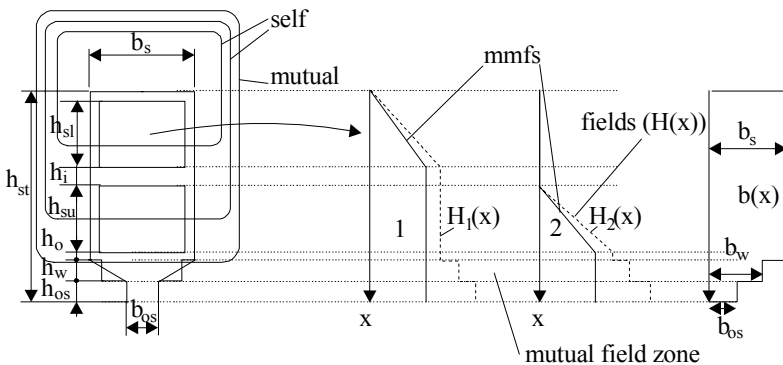


Figure 6.7 Two-layer rectangular semiclosed slots: leakage field

$$H_1(x) + H_2(x) = \begin{cases} \frac{n_{cl} I}{b_s} \cdot \frac{x}{h_{sl}}; & \text{for } 0 < x < h_{sl} \\ \frac{n_{cl} I}{b_s} & \text{for } h_{sl} < x < h_{sl} + h_i \\ \frac{n_{cl} I}{b_s} + \frac{n_{cu} I \cos \gamma_k}{b_s} \frac{(x - h_{sl} - h_i)}{h_{su}}; & \text{for } h_{sl} + h_i < x < h_{sl} + h_i + h_{su} \\ \frac{n_{cl} I}{b_i} + \frac{n_{cu} I \cos \gamma_k}{b_i}; & \text{for } x > h_{sl} + h_i + h_{su} \end{cases} \quad (6.23)$$

with $b_i = b_w$ or b_{os}

The phase shift between currents in lower and upper layer coils of slot K is γ_K and n_{cl} , n_{cu} are the number of turns of the two coils. Adding up the effect of all slots per phase (1/3 of total number of slots), the average slot leakage inductance per phase L_{sl} is obtained.

$$\lambda_{\text{sk}} = \frac{L_{\text{sl}}}{\mu_0(2n_c)^2 L_c} = \frac{1}{4} \left[\frac{(h_{\text{sl}} + h_{\text{su}} \cos^2 \gamma_k)}{3b_s} + \frac{h_{\text{su}}}{b_s} + \frac{h_{\text{su}} \cos \gamma_k}{b_s} + \frac{h_i}{b_s} + (1 + \cos \gamma)^2 \left(\frac{h_o}{b_s} + \frac{h_w}{b_w} + \frac{h_{\text{os}}}{b_{\text{os}}} \right) \right] \quad (6.24)$$

Other realistic rectangular slot shapes for large power IMs (Figure 6.8) may also be handled via (6.24) with minor adaptations.

$$(\lambda_{sk})_{h_{su}=h_{sl}=h_s'}^{\gamma_k=0} = \frac{2h_s'}{3b_s} + \frac{h_i}{4b_s} + \frac{h_0}{b_s} + \frac{h_w}{b_w} + \frac{h_{os}}{b_{os}} \quad (6.25)$$

Figure 6.8 Typical high power IM stator slots

6.5. ROUNDED SHAPE SLOT LEAKAGE INDUCTANCE/TWO LAYERS

Although the integral in (6.2) does not have exact analytical solutions for slots with rounded corners, or purely circular slots (Figure 6.9), so typical to low-power IMs, some approximate solutions have become standard for design purposes:

- For slots a.) in Figure 6.9,

$$\lambda_{s,r} \approx \frac{2h_{s,r}K_1}{3(b_1 + b_2)} + \left(\frac{h_{os,r}}{b_{os,r}} + \frac{h_o}{b_1} - \frac{b_{os,r}}{2b_1} + 0.785 \right) K_2 \quad (6.26)$$

with

$$\begin{aligned} K_2 &\approx \frac{1+3\beta_y}{4}; \quad \text{for } \frac{2}{3} \leq \beta_y = \frac{y}{\tau} \leq 1 \\ K_2 &\approx \frac{6\beta_y - 1}{4}; \quad \text{for } \frac{1}{3} \leq \beta_y \leq \frac{2}{3} \\ K_2 &\approx \frac{3(2-\beta_y)+1}{4}; \quad \text{for } 1 \leq \beta_y \leq 2 \\ K_1 &\approx \frac{1}{4} + \frac{3}{4} K_2 \end{aligned} \quad (6.27)$$

- For slots b.),

$$\lambda_{s,r} \approx \frac{2h_{s,r}K_1}{3(b_1 + b_2)} + \left(\frac{h_{os,r}}{b_{os,r}} + \frac{h_o}{b_1} + \frac{3h_w}{b_1 + 2b_{os,r}} \right) K_2 \quad (6.28)$$

- For slots c.),

$$\lambda_r = 0.785 - \frac{b_{or}}{2b_1} + \frac{h_{or}}{b_{or}} \approx 0.66 + \frac{h_{or}}{b_{or}} \quad (6.29)$$

- For slots d.),

$$\lambda_r = \frac{h_r}{3b_1} \left(1 - \frac{\pi b_1^2}{8A_b} \right)^2 + 0.66 - \frac{h_{or}}{2b_1} + \frac{h_{or}}{b_{or}} \quad (6.30)$$

where A_b is the bar cross section.

If the slots in Figure 6.9c, d are closed ($h_o = 0$) (Figure 6.9e) the terms h_{or}/b_{or} in Equations (6.29, 6.30) may be replaced by a term dependent on the bar current which saturates the iron bridge.

$$\frac{h_{or}}{b_{or}} \rightarrow \approx 0.3 + 1.12h_{or} \frac{10^3}{I_b^2}; \quad I_b > 5b_1 10^3; \quad b_1 \text{ in [m]} \quad (6.31)$$

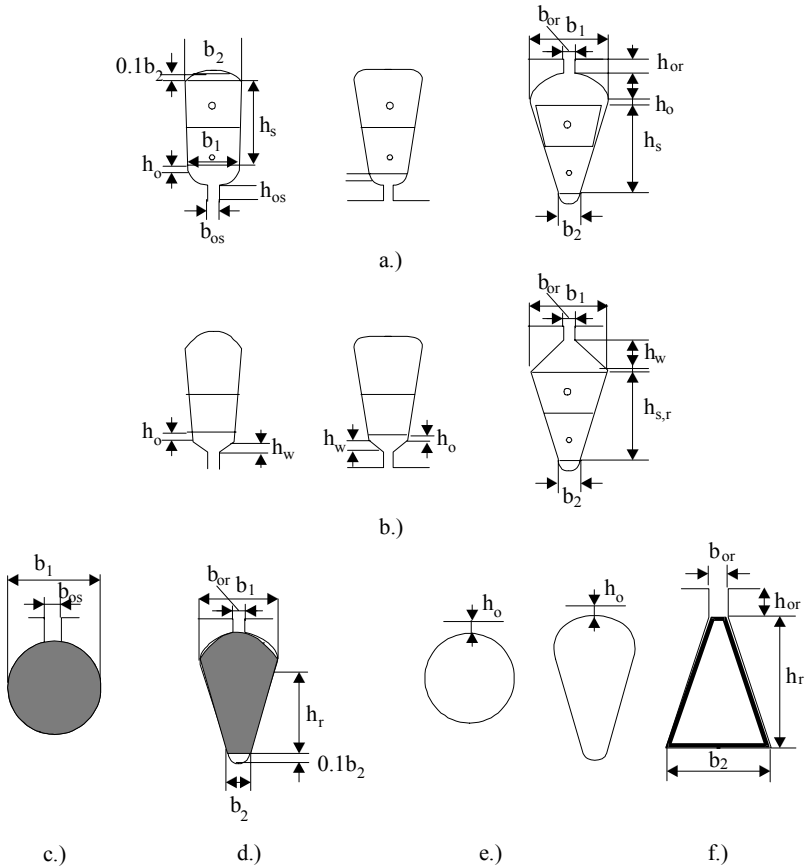


Figure 6.9 Rounded slots: oval, trapezoidal, and round

This is only an empirical approximation for saturation effects in closed rotor slots, potentially useful for very preliminary design purposes.

For the trapezoidal slot (Figure 6.9f), typical for deep rotor bars in high power IMs, by conformal transformations, the slot permeance is, approximately [3]

$$\lambda_r = \frac{1}{\pi} \left[\ln \frac{\left(\frac{b_2}{b_{or}} \right)^2 - 1}{4 \frac{b_2}{b_{or}}} + \frac{\left(\frac{b_2}{b_{or}} \right)^2 + 1}{\frac{b_2}{b_{or}}} \cdot \ln \frac{\frac{b_2}{b_{or}} - 1}{\frac{b_2}{b_{or}} + 1} \right] + \frac{h_{or}}{b_{or}} \quad (6.32)$$

The term in square brackets may be used to calculate the geometrical permeance of any trapezoidal slot section (wedge section, for example).

Finally for stator (and rotors) with radial ventilation ducts (channels) additional slot leakage terms have to be added. [8]

For more complicated rotor cage slots used in high skin effect (low starting current, high starting torque) applications, where the skin effect is to be considered, pure analytical solutions are hardly feasible, although many are still in industrial use. Realistic computer-aided methods are given in Chapter 8.

6.6 ZIG-ZAG AIRGAP LEAKAGE INDUCTANCES

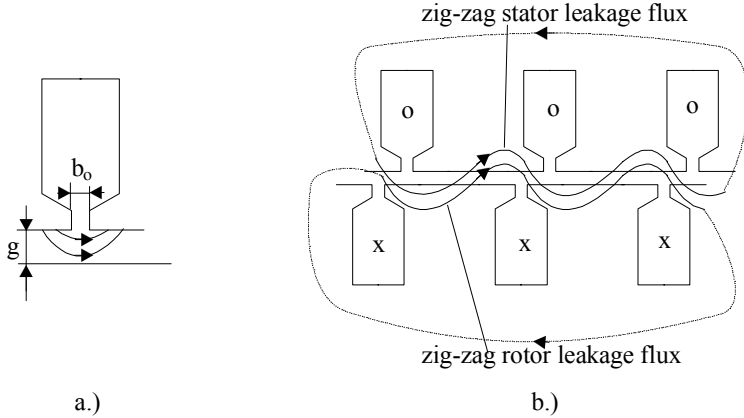


Figure 6.10 Airgap a.) and zig-zag b.) leakage fields

The airgap flux does not reach the other slotted structure (Figure 6.10a) while the zig-zag flux “snakes” out through the teeth around slot openings.

In general, they may be treated together either by conformal transformation or by FEM. From conformal transformations, the following approximation is given for the geometric permeance $\lambda_{zs,r}$ [3]

$$\lambda_{zs,r} \approx \frac{5gK_c / b_{os,r}}{5 + 4gK_c / b_{os,r}} \cdot \frac{3\beta_y + 1}{4} < 1.0; \beta_y = 1 \text{ for cage rotors} \quad (6.32)$$

The airgap zig-zag leakage inductance per phase in stator-rotor is

$$L_{zls} \approx 2\mu_0 \frac{W_l L_e}{p_l q} \lambda_{zs,r} \quad (6.33)$$

In [4], different formulas are given

$$L_{zls} = L_{lm} \cdot \frac{\pi^2 p_1^2}{12 N_s^2} \left[1 - \frac{a(1+a)(1-K')}{2K'} \right] \quad (6.34)$$

for the stator, and

$$L_{zlr} = L_{lm} \cdot \frac{\pi^2 p_1^2}{12 N_s^2} \left[\frac{N_s^2}{N_r^2} - \frac{a(1+a)(1-K')}{2K'} \right] \quad (6.35)$$

for the rotor with $K' = 1/K_c$, $a = b_{ls,r}/\tau_{s,r}$, $\tau_{s,r}$ = stator (rotor) slot pitch, $b_{ls,r}$ —stator (rotor) tooth-top width.

It should be noticed that while expression (6.32) is dependent only on the airgap/slot opening, in (6.34) and (6.35) the airgap enters directly the denominator of L_{lm} (magnetization inductance) and, in general, (6.34) and (6.35) includes the number of slots of stator and rotor, N_s and N_r .

As the term in parenthesis is a very small number an error here will notably “contaminate” the results. On the other hand, iron saturation will influence the zig-zag flux path, but to a much lower extent than the magnetization flux as the airgap is crossed many times (Figure 6.10b). Finally, the influence of chorded coils is not included in (6.34) to (6.35). We suggest the use of an average of the two expressions (6.33) and (6.34 or 6.35).

In Chapter 7 we revisit this subject for heavy currents (at standstill) including the actual saturation in the tooth tops.

Example 6.2. Zig-zag leakage inductance

For the machine in Example 6.1, with $g = 0.5 \cdot 10^{-3}$ m, $b_{os} = 6$ g, $b_{or} = 3$ g, $K_c = 1.32$, $L_{lm} = 0.1711$ H, $p_1 = 2$, $N_s = 36$ stator slots, $N_r = 30$ rotor slots, stator bore $D_i = 0.102$ m, $B_y = y/\tau = 8/9$ (chorded coils), and $W_1 = 300$ turns/phase, let us calculate the zig-zag leakage inductance both from (6.32 – 6.33) and (6.34 – 6.35).

Solution. Let us prepare first the values of $K' = 1/K_c = 1/1.32 = 0.7575$.

$$a_{s,r} = \frac{\tau_{s,r} - b_{os,r}}{\tau_{s,r}} =$$

$$= \begin{cases} 1 - \frac{b_{os} N_s}{\pi D_i} = 1 - \frac{6 \cdot 0.5 \cdot 10^{-3} \cdot 36}{\pi \cdot 0.102} = 0.6628 \\ 1 - \frac{b_{or} N_r}{\pi (D_i - 2g)} = 1 - \frac{3 \cdot 0.5 \cdot 10^{-3} \cdot 30}{\pi \cdot 0.101} = 0.858 \end{cases} \quad (6.37)$$

$$\text{From (6.32),} \quad \lambda_{Zs} = \frac{\frac{5 \cdot 0.5 \cdot 10^{-3} \cdot 1.32}{6 \cdot 0.1 \cdot 1.32}}{5 + \frac{6 \cdot 0.1 \cdot 10^{-3}}{4 \cdot 0.1 \cdot 1.32}} = 0.187 \quad (6.38)$$

$$\lambda_{Zr} = \frac{\frac{5 \cdot 0.1 \cdot 10^{-3} \cdot 1.32}{3 \cdot 0.1 \cdot 10^{-3}}}{5 + \frac{4 \cdot 0.1 \cdot 1.32}{3 \cdot 0.1}} = 0.101 \quad (6.39)$$

The zig-zag inductances per phase $L_{zls,r}$ are calculated from (6.33).

$$L_{zls,r} = \begin{cases} \frac{2 \cdot 1.256 \cdot 10^{-6} \cdot 300^2 \cdot 0.12}{2 \cdot 3} \cdot 0.187 = 8.455 \cdot 10^{-4} \text{ H} \\ \frac{2 \cdot 1.256 \cdot 10^{-6} \cdot 300^2 \cdot 0.12}{2 \cdot 3} \cdot 0.101 = 4.566 \cdot 10^{-4} \text{ H} \end{cases} \quad (6.40)$$

Now from (6.34) – (6.35),

$$L_{zls} = 0.1711 \cdot \frac{\pi^2 2^2}{12 \cdot 36^2} \left[1 - \frac{0.6628(1 + 0.6628)(1 - 0.7575)}{2 \cdot 0.7575} \right] = 3.573 \cdot 10^{-4} \text{ H} \quad (6.41)$$

$$L_{zlr} = 0.1711 \cdot \frac{\pi^2 2^2}{12 \cdot 36^2} \left[\left(\frac{36}{30} \right)^2 - \frac{0.858(1 + 0.858)(1 - 0.7575)}{2 \cdot 0.7575} \right] = 3.723 \cdot 10^{-4} \text{ H} \quad (6.42)$$

All values are small in comparison to $L_{lm} = 1711 \cdot 10^{-4} \text{ H}$, but there are notable differences between the two methods. In addition it may be inferred that the zig-zag flux leakage also includes the differential leakage flux.

6.7. END-CONNECTION LEAKAGE INDUCTANCE

As seen in [Figure 6.11](#), the three-dimensional character of end connection field makes the computation of its magnetic energy and its leakage inductance per phase a formidable task.

Analytical field solutions need bold simplifications. [5]. Biot-Savart inductance formula [6] and 3D FEM have all been also tried for particular cases.

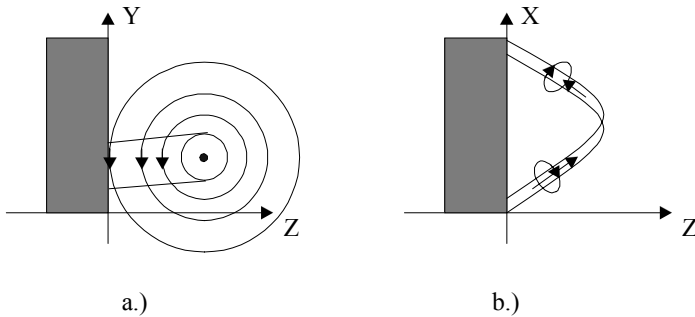


Figure 6.11 Three-dimensional end connection field

Some widely used expressions for the end connection geometrical permeances are as follows:

- Single-layer windings (with end turns in two “stores”).

$$\lambda_{es,r} = 0.67 \cdot \frac{q_{s,r}}{L_e} (l_{es,r} - 0.64\tau) \quad (6.43)$$

- Single-layer windings (with end connections in three “stores”).

$$\lambda_{es,r} = 0.47 \cdot \frac{q_{s,r}}{L_e} (l_{es,r} - 0.64\tau) \quad (6.44)$$

- Double-layer (or single-layer) chain windings.

$$\lambda_{es,r} = 0.34 \cdot \frac{q_{s,r}}{L_e} (l_{es,r} - 0.64y) \quad (6.45)$$

with $q_{s,r}$ —slots/pole/phase in the stator/rotor, L_e —stack length, y —coil throw, $l_{es,r}$ —end connection length per motor side.

For cage rotors,

- With end rings attached to the rotor stack.

$$\lambda_{ei} \approx \frac{2 \cdot 3 \cdot D_{ir}}{4N_r L_e \sin^2 \left(\frac{\pi p_l}{N_r} \right)} l_g 4.7 \frac{D_{ir}}{a + 2b} \quad (6.46)$$

- With end rings distanced from the rotor stack:

$$\lambda_{ei} \approx \frac{2 \cdot 3 \cdot D_i}{4N_r L_e \sin^2 \left(\frac{\pi p_l}{N_r} \right)} l_g 4.7 \frac{D_i}{2(a + b)} \quad (6.47)$$

with a and b the ring axial and radial dimensions, and D_{ir} the average end ring diameter.

6.8. SKEWING LEAKAGE INDUCTANCE

In Chapter 4 on windings, we did introduce the concept of skewing (uncompensated) rotor mmf, variable along axial length, which acts along the main flux path and produces a flux which may be considered of leakage character. Its magnetic energy in the airgap may be used to calculate the equivalent inductance.

This skewing inductance depends on the local level of saturation of rotor and stator teeth and cores, and acts simultaneously with the magnetization flux, which is phase shifted with an angle dependent on axial position and slip. We will revisit this complex problem in Chapter 8. In a first approximation, we can make use of the skewing factor K_{skew} and define $L_{skew,r}$ as

$$L_{skew,r} = (1 - K_{skew}^2) L_{lm} \quad (6.48)$$

$$K_{\text{skew}} = \frac{\sin \alpha_{\text{skew}}}{\alpha_{\text{skew}}}; \quad \alpha_{\text{skew}} = \frac{c}{\tau} \frac{\pi}{2} \quad (6.49)$$

As this inductance “does not act” when the rotor current is zero, we feel it should all be added to the rotor.

Finally, the total leakage inductance of stator (rotor) is:

$$L_{\text{ls}} = L_{\text{dls}} + L_{\text{zls}} + L_{\text{sls}} + L_{\text{els}} = 2\mu_0 \frac{W_l L_e}{pq} \sum \lambda_{\text{si}} \quad (6.50)$$

$$L_{\text{lr}} = L_{\text{dlr}} + L_{\text{zlr}} + L_{\text{slr}} + L_{\text{elr}} + L_{\text{skew,r}} \quad (6.51)$$

Although we discussed rotor leakage inductance as if basically reduced to the stator, this operation is due later in this chapter.

6.9. ROTOR BAR AND END RING EQUIVALENT LEAKAGE INDUCTANCE

The differential leakage L_{dlr} , zig-zag leakage L_{zlr} and $L_{\text{skew,r}}$ in (6.51) are already considered in stator terms (reduced to the stator). However, the terms L_{slr} and L_{elr} related to rotor slots (bar) leakage and end connection (end ring) leakage are not clarified enough.

To do so, we use the equivalent bar (slot) leakage inductance expression L_{be} ,

$$L_{\text{be}} = L_{\text{b}} + 2L_{\text{i}} \quad (6.52)$$

where L_{b} is the slot (bar) leakage inductance and L_{i} is the end ring (end connection) segment leakage inductance (Figure 6.13)

Using (6.18) for one slot with $n_c = 1$ conductors yields:

$$L_{\text{b}} = \mu_0 L_{\text{b}} \lambda_{\text{b}} \quad (6.53)$$

$$L_{\text{ei}} = \mu_0 L_{\text{i}} \lambda_{\text{er}} \quad (6.54)$$

where λ_{b} is the slot geometrical permeance, for a single layer in slot as calculated in paragraphs 6.4 and 6.5 for various slot shapes and λ_{er} is the geometrical permeance of end ring segment as calculated in (6.46) and (6.47).

Now all it remains to obtain L_{slr} and L_{elr} in (6.51), is to reduce L_{be} in (6.52) to the stator. This will be done in paragraph 6.13.

6.10. BASIC PHASE RESISTANCE

The stator resistance R_s is plainly

$$R_s = \rho_{\text{Co}} l_c \frac{W_a}{a} \frac{1}{A_{\text{cos}}} K_R \quad (6.55)$$

where ρ_{Co} is copper resistivity ($\rho_{Co} = 1.8 \cdot 10^{-8} \Omega m$ at $25^\circ C$), l_c the turn length:

$$l_c = 2L_e + 2b + 2l_{ec} \quad (6.56)$$

b —axial length of coil outside the core per coil side; l_{ec} —end connection length per stack side, L_e —stack length; A_{cos} —actual conductor area, a —number of current paths, W_a —number of turns per path. With $a = 1$, $W_a = W_1$ turns/phase. K_R is the ratio between the a.c. and d.c. resistance of the phase resistance.

$$K_R = \frac{R_{sac}}{R_{sdc}} \quad (6.57)$$

For 50 Hz in low and medium power motors, the conductor size is small with respect to the field penetration depth δ_{Co} in it.

$$\delta_{Co} = \sqrt{\frac{\rho_{Co}}{\mu_0 \pi f_1}} < d_{Co} \quad (6.58)$$

In other words, the skin effect is negligible. However, in high frequency (speed) special IMs, this effect may be considerable unless many thin conductors are transposed (as in litz wire). On the other hand, in large power motors, there are large cross sections (even above 60 mm^2) where a few elementary conductors are connected in parallel, even transposed in the end connection zone, to reduce the skin effect (Figure 6.12). For K_R expressions, check Chapter 9. For wound rotors, (6.55) is valid.

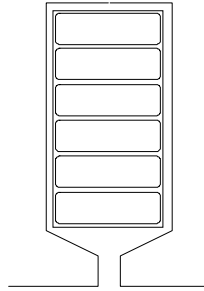


Figure 6.12 Multi-conductor single-turn coils for high power IMs

6.11. THE CAGE ROTOR RESISTANCE

The rotor cage geometry is shown in Figure 6.13.

Let us denote by R_b the bar resistance and by R_i the ring segment resistance:

$$R_b = \rho_b \frac{l_b}{A_b}; \quad R_i = \rho_i \frac{l_i}{A_i}; \quad A_i = a \cdot b \quad (6.59)$$

As the emf in rotor bars is basically sinusoidal, the phase shift α_{er} between neighbouring bars emf is

$$\alpha_{er} = \frac{2\pi p_1}{N_r} \quad (6.60)$$

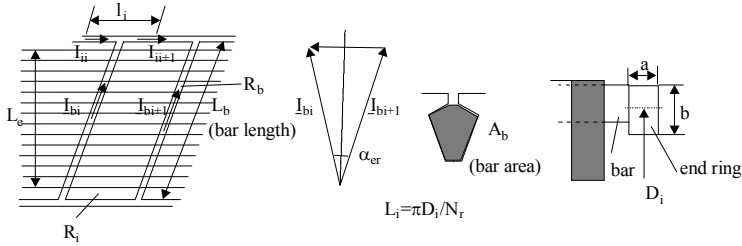


Figure 6.13. Rotor cage geometry

The current in a bar, I_{bi} , is the difference between currents in neighboring ring segments I_{ii} and I_{ii+1} (Figure 6.13).

$$I_{bi} = I_{ii+1} - I_{ii} = 2I_i \sin(\alpha_{er} / 2) \quad (6.61)$$

The bar and ring segments may be lumped into an equivalent bar with a resistance R_{be} .

$$R_{be} I_b^2 = R_b I_b^2 + 2R_i I_i^2 \quad (6.62)$$

With (6.61), Equation (6.62), leads to

$$R_{be} = R_b + \frac{R_i}{2 \sin^2 \left(\frac{\pi p_1}{N_r} \right)} \quad (6.63)$$

When the number of rotor slot/pole pair is small or fractionary (6.49) becomes less reliable.

Example 6.3. Bar and ring resistance

For $N_r = 30$ slots per rotor $2p_1 = 4$, a bar current $I_b = 1000$ A with a current density $j_{cob} = 6$ A/mm² in the bar and $j_{coi} = 5$ A/mm² in the end ring, the average ring diameter $D_{ir} = 0.15$ m, $l_b = 0.14$ m, let us calculate the bar, ring cross section, the end ring current, and bar and equivalent bar resistance.

Solution

The bar cross-section A_b is

$$A_b = \frac{I_b}{j_{cob}} = 1000 / 6 = 166 \text{ mm}^2 \quad (6.64)$$

The current in the end ring I_i (from (6.61)) is:

$$I_i = \frac{I_b}{2 \sin \alpha_{er} / 2} = \frac{1000}{2 \sin \frac{2\pi}{30}} = 2404.86 \text{ A} \quad (6.65)$$

In general, the ring current is greater than the bar current. The end ring cross-section A_i is

$$A_i = \frac{I_i}{j_{coi}} = \frac{2404.86}{5} = 480.97 \text{ mm}^2 \quad (6.66)$$

The end ring segment length,

$$L_i = \frac{\pi D_{ir}}{N_r} = \frac{\pi \cdot 0.150}{30} = 0.0157 \text{ m} \quad (6.67)$$

The end ring segment resistance R_i is

$$R_i = \rho_{Al} \frac{L_i}{A_i} = \frac{3 \cdot 10^{-8} \cdot 0.0157}{480.97 \cdot 10^{-6}} = 0.9 \cdot 10^{-6} \Omega \quad (6.68)$$

The rotor bar resistance R_b is

$$R_b = \rho_{Al} \frac{L_b}{A_b} = \frac{3 \cdot 10^{-8} \cdot 0.15}{166 \cdot 10^{-6}} = 2.7 \cdot 10^{-5} \Omega \quad (6.69)$$

Finally, (from (6.63)), the equivalent bar resistance R_{be} is

$$R_{be} = 2.7 \cdot 10^{-5} + \frac{0.9 \cdot 10^{-6}}{2 \sin \frac{2\pi}{30}} = 3.804 \cdot 10^{-5} \Omega \quad (6.70)$$

As we can see from (6.70), the contribution of the end ring to the equivalent bar resistance is around 30%. This is more than a rather typical proportion. So far we did consider that the distribution of the currents in the bars and end rings are uniform. However, there are a.c. currents in the rotor bars and end rings. Consequently, the distribution of the current in the bar is not uniform and depends essentially on the rotor frequency $f_2 = sf_1$.

Globally, the skin effect translates into resistance and slot leakage correction coefficients $K_R > 1$, $K_x < 1$ (see Chapter 9).

In general, the skin effect in the end rings is neglected. The bar resistance R_b in (6.69) and the slot permeance λ_b in (6.53) are modified to

$$R_b = \rho_{Al} \frac{L_b}{A_b} K_R \quad (6.71)$$

$$L_b = \mu_0 L_b \lambda_b K_x \quad (6.72)$$

6.12. SIMPLIFIED LEAKAGE SATURATION CORRECTIONS

Further on, for large values of currents (large slips), the stator (rotor) tooth tops tend to be saturated by the slot leakage flux (Figure 6.14).

Neglecting the magnetic saturation along the tooth height, we may consider that only the tooth top saturates due to the slot neck leakage flux produced by the entire slot mmf.

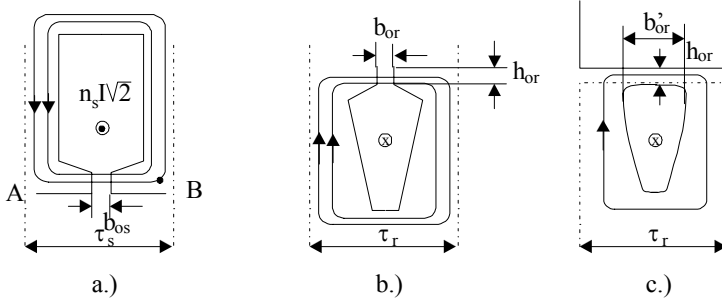


Figure 6.14 Slot neck leakage flux

a.) stator semiclosed slot b.) rotor semiclosed slot c.) rotor closed slot

A simple way to account for this leakage saturation that is used widely in industry consists of increasing the slot opening $b_{os,r}$ by the tooth top length t_t divided by the relative iron permeability μ_{rel} within it.

$$\begin{aligned} t_t &= \tau_{s,r} - b_{os,r} \\ b'_{os,r} &= b_{os,r} + \frac{t_t}{\mu_{rel}} \end{aligned} \quad (6.73)$$

For the closed slot (Figure 6.14), we may use directly (6.31). To calculate μ_{rel} , we apply the Ampere's and flux laws.

$$\begin{aligned} B_t &= \mu_0 H_0 \\ n_s I \sqrt{2} &= t_t H_t + H_0 b_{os,r} \end{aligned} \quad (6.74)$$

$$\mu_0 n_s I \sqrt{2} = \mu_0 t_t H_t + B_t b_{os,r}; \quad B_0 = B_t \quad (6.75)$$

We have to add the lamination magnetization curve, $B_t(H_t)$, to (6.75) and intersect them (Figure 6.15) to find B_t and H_t , and, finally,

$$\mu_{rel} = \frac{B_t}{\mu_0 H_t} \quad (6.76)$$

H_0 —magnetic field in the slot neck.

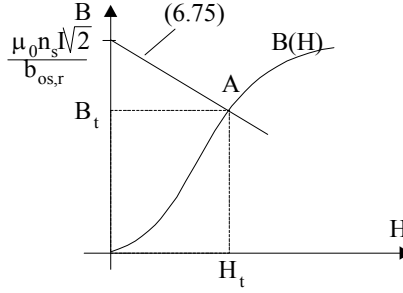


Figure 6.15 Tooth top flux density B_t

Given the slot mmf $n_s I \sqrt{2}$ (the current, in fact), we may calculate iteratively μ_{rel} (6.76) and, from (6.73), the corrected $b_{os,r}$ ($b'_{os,r}$) then to be used in geometrical slot permeance calculations.

As expected, for open slots, the slot leakage saturation is negligible. Simple as it may seem, a second iteration process is required to find the current, as, in general, a voltage type supply is used to feed the IM.

It may be inferred that both differential and skewing leakage are influenced by saturation, in the sense of reducing them. For the former, we already introduced the partial teeth saturation factor $K_{st} = K_{sd}$ to account for it (6.14). For the latter, in (6.48), we have assumed that the level of saturation is implicitly accounted for L_{lm} (that is, it is produced by the magnetization current in the machine). In reality, for large rotor currents, the skewing rotor mmf field, dependent on rotor current and axial position along stack, is quite different from that of the main flux path, at standstill.

However, to keep the formulas simple, Equation (6.73) has become rather standard for design purposes.

Zig-zag flux path tends also to be saturated at high currents. Notice that the zig-zag flux path occupies the teeth tops (Figure 6.10b). This aspect is conventionally neglected as the zig-zag leakage inductance tends to be a small part of total leakage inductance.

So far, we considered the leakage saturation through simple approximate correction factors, and treated skin effects only for rectangular coils. In Chapter 9 we will present comprehensive methods to treat these phenomena for slots of general shapes. Such slot shapes may be the result of design optimization methods based on various cost (objective) functions such as high starting torque, low starting current, large peak torque, etc.

6.13. REDUCING THE ROTOR TO STATOR

The main flux paths embrace both the stator and rotor slots passing through an airgap. So the two emfs per phase $\underline{E}_1$ (Chapter 5.10) and $\underline{E}_2$ are

$$\underline{E}_1 = -j\pi\sqrt{2}W_1K_{q1}K_{y1}f_1\underline{\Phi}_1; \quad \Phi_1 = \frac{2}{\pi}B_{g1}\tau L_e \quad (6.77)$$

$$\underline{E}_2 = -j\pi\sqrt{2}W_2K_{q2}K_{y2}K_{e2}f_1S\underline{\Phi}_1 \quad (6.77)$$

We have used phasors in (6.77) and (6.78) as E_1, E_2 are sinusoidal in time (only the fundamental is accounted for).

Also in (6.78), E_2 is already calculated as “seen from the stator side,” in terms of frequency because only in this case Φ_1 is the same in both stator and rotor phases. In reality the flux in the rotor varies at Sf_1 frequency, while in the stator at f_1 frequency. The amplitude is the same and, with respect to each other, they are at stall.

Now we may proceed as for transformers by dividing the two equations to obtain

$$\frac{E_1}{E_2} = \frac{W_1K_{w1}}{W_2K_{w2}} = K_e = \frac{V_r'}{V_r} \quad (6.79)$$

K_e is the voltage reduction factor to stator.

Eventually the actual and the stator reduced rotor mmfs should be identical:

$$\frac{m_1\sqrt{2}K_{w1}I_r'W_1}{\pi p_1} = \frac{m_2\sqrt{2}K_{w2}I_rW_2}{\pi p_1} \quad (6.80)$$

From (6.80) we find the current reduction factor K_i .

$$K_i = \frac{I_r'}{I_r} = \frac{m_2K_{w2}W_2}{m_1K_{w1}W_1} \quad (6.81)$$

For the rotor resistance and reactance equivalence, we have to conserve the conductor losses and leakage field energy.

$$m_1R_r'I_r'^2 = m_2R_rI_r^2 \quad (6.82)$$

$$m_1\frac{L_{rl}'}{2}I_r'^2 = m_2\frac{L_{rl}}{2}I_r^2 \quad (6.83)$$

So,
$$R_r' = R_r \frac{m_2}{m_1} \frac{1}{K_i^2}; \quad L_{rl}' = L_{rl} \frac{m_2}{m_1} \frac{1}{K_i^2} \quad (6.84)$$

For cage rotors, we may still use (6.81) and (6.84) but with $m_2 = N_r$, $W_2 = 1/2$, and $K_{w2} = K_{skew}$ (skewing factor); that is, each bar (slot) represents a phase.

$$R_r' = R_{be} \frac{12K_{wl}^2 W_1^2}{N_r K_{skew}^2} \quad (6.85)$$

$$L_{be}' = L_{be} \frac{12K_{wl}^2 W_1^2}{N_r K_{skew}^2} \quad (6.86)$$

Notice that only the slot and ring leakage inductances in (6.86) have to be reduced to the stator in L_{rl} (6.81).

$$L_{rl}' = (L_{dlr} + L_{zlr} + L_{skew,r}) + L_{be}' \quad (6.87)$$

Example 6.4. Let us consider an IM with $q = 3$, $p_1 = 2$, ($N_s = 36$ slots), $W_1 = 300$ turns/phase, $K_{wl} = 0.965$, $N_r = 30$ slots, one stator slot pitch rotor skewing ($c/\tau = 1/3q$) ($K_{skew} = 0.9954$), and the rotor bar (slot) and end ring cross sections are rectangular ($h_r/b_1 = 2/1$), $h_{or} = 1.5 \cdot 10^{-3}$ m, and $b_{or} = 1.5 \cdot 10^{-3}$ m, $L_e = 0.12$ m. Let us find the bar ring resistance and leakage inductance reduced to the stator.

Solution

From Example 6.3, we take $D_{ir} = 0.15$ m, $A_i = a \times b = 481 \text{ mm}^2$ for the end ring. We may assume $a/b = 1/2$ and, thus, $a = 15.51 \cdot 10^{-3}$ m and $b = 31$ mm.

The bar cross-section in example 6.3 is $A_b = 166 \text{ mm}^2$.

$$b_1 = \sqrt{\frac{A_b}{(h_r/b_1)}} = \sqrt{\frac{166}{2}} \approx 9.1 \text{ mm}; \quad h_r = 18.2 \text{ mm} \quad (6.88)$$

From example 6.3 we already know the equivalent bar resistance (6.70) $R_{be} = 3.804 \cdot 10^{-5} \Omega$.

Also from (6.19), the rounded geometrical slot permeance λ_r is,

$$\lambda_r = \lambda_{bar} \approx \frac{h_r}{3b_1} + \frac{h_{or}}{b_{or}} = \frac{18.2}{3 \cdot 9.1} + \frac{1.5}{1.5} = 1.66 \quad (6.89)$$

From (6.46), the end ring segment geometrical permeance λ_{ei} may be found.

$$\begin{aligned} \lambda_{ei} &= \frac{2.3D_{ir}}{4N_r L_e \sin^2 \frac{\pi P_1}{N_r}} \lg \frac{4.7D_i}{a+2b} = \\ &= \frac{2.3 \cdot 0.15}{4 \cdot 30 \cdot 0.12 \cdot \sin^2 \frac{\pi 2}{18}} \lg \frac{4.7 \cdot 0.15}{(1.5 + 2 \cdot 31) \cdot 10^{-3}} = 0.1964 \end{aligned} \quad (6.90)$$

with $L_i = \pi D_i/N_r = \pi \cdot 150/30 = 15.7 \cdot 10^{-3}$ m.

Now from (6.39) and (6.40), the bar and end ring leakage inductances are

$$\begin{aligned} L_b &= \mu_0 l_b \lambda_{\text{bar}} = 1.256 \cdot 10^{-6} \cdot 0.14 \cdot 1.66 = 0.292 \cdot 10^{-6} \text{ H} \\ L_{li} &= \mu_0 l_i \lambda_{ei} = 1.256 \cdot 10^{-6} \cdot 0.0157 \cdot 1.964 = 0.3872 \cdot 10^{-8} \text{ H} \end{aligned} \quad (6.91)$$

The equivalent bar leakage inductance L_{be} (6.52) is written:

$$L_{be} = L_b + 2L_{ei} = 0.292 \cdot 10^{-6} + 2 \cdot 0.3872 \cdot 10^{-8} = 0.2997 \cdot 10^{-6} \text{ H} \quad (6.92)$$

From (6.85) and (6.86) and we may now obtain the rotor slot (bar) and end ring equivalent resistance and leakage inductance reduced to the stator,

$$R_r' = R_{be} \frac{12K_{wl}^2 W_1^2}{N_r K_{\text{skew}}^2} = \frac{3.804 \cdot 10^{-5} \cdot 12 \cdot 0.965^2 \cdot 300^2}{30 \cdot 0.99^2} = 1.28 \Omega \quad (6.93)$$

$$L_{be}' = L_{be} \frac{12K_{wl}^2 W_1^2}{N_r K_{\text{skew}}^2} = 0.2997 \cdot 10^{-6} \cdot 0.36 \cdot 10^{-5} = 1.008 \cdot 10^{-2} \text{ H} \quad (6.94)$$

As a bonus, knowing from Example 6.3 that the magnetization inductance $L_{1m} = 0.1711 \text{ H}$, we may calculate the rotor skewing leakage inductance (from 6.48), which is already reduced to the stator because it is a fraction of L_{1m} ,

$$L_{\text{skew},r} = (1 - K_{\text{skew}}^2) \cdot L_{1m} = (1 - 0.9954^2) \cdot 0.1711 = 1.57 \cdot 10^{-3} \text{ H} \quad (6.95)$$

In this particular case the skewing leakage is more than 6 times smaller than the slot (bar) and end ring leakage inductance. Notice also that $L_{be}'/L_{1m} = 0.059$, a rather practical value.

6.14. SUMMARY

- Besides main path lines which embrace stator and rotor slots, and cross the airgap to define the magnetization inductance L_{1m} , there are leakage fields that encircle either the stator or the rotor conductors.
- The leakage fields are divided into differential leakage, zig-zag leakage, slot leakage, end-turn leakage, and skewing leakage. Their corresponding inductances are calculated from their stored magnetic energy.
- Step mmf harmonic fields through airgap induce emfs in the stator, while the space rotor mmf harmonics do the same. These space harmonics produced emfs have the supply frequency and this is why they are considered of leakage type. Magnetic saturation of stator and rotor teeth reduces the differential leakage.
- Stator differential leakage is minimum for $y/\tau = 0.8$ coil throw and decreases with increasing q (slots/pole/phase).
- A quite general graphic-based procedure, valid for any practical winding, is used to calculate the differential leakage inductance.
- The slot leakage inductance is based on the definition of a geometrical permeance λ_s dependent on the aspect ratio. In general, $\lambda_s \in (0.5 \text{ to } 2.5)$ to

keep the slot leakage inductance within reasonable limits. For a rectangular slot, some rather simple analytical expressions are obtained even for double-layer windings with chorded, unequal coils.

- The saturation of teeth tops due to high currents at large slips reduces λ_s and it is accounted for by a pertinent increase of slot opening which is dependent on stator (rotor) current (mmf).
- The zig-zag leakage flux lines of stator and rotor mmf snake through airgap around slot openings and close out through the two back cores. At high values of currents (large slip values), the zig-zag flux path, mainly the teeth tops, tends to saturate because the combined action of slot neck saturation and zig-zag mmf contribution.
- The rotor slot skewing leads to the existence of a skewing (uncompensated) rotor mmf which produces a leakage flux along the main paths but its maximum is phase shifted with respect to the magnetization mmf maximum and is dependent on slip and position along the stack length. As an approximation only, a simple analytical expression for an additional rotor-only skewing leakage inductance $L_{\text{skew},r}$ is given.
- Leakage path saturation reduces the leakage inductance.
- The a.c. stator resistance is higher than the d.c. because of skin effect, accounted for by a correction coefficient K_R , calculated in Chapter 9. In most IMs, even at higher power but at 50 (60) Hz, the skin effect in the stator is negligible for the fundamental. Not so for current harmonics present in converter-fed IMs or in high-speed (high-frequency) IMs.
- The rotor bar resistance in squirrel cage motors is, in general, increased notably by skin effect, for rotor frequencies $f_2 = Sf_1 > (4 - 5)\text{Hz}$; $K_R > 1$.
- The skin effect also reduces the slot geometrical permeance ($K_x < 1$) and, finally, also the leakage inductance of the rotor.
- The rotor cage (or winding) has to be reduced to the stator to prepare the rotor resistance and leakage inductance for utilization in the equivalent circuit of IMs. The equivalent circuit is widely used for IM performance computation.
- Accounting for leakage saturation and skin effect in a comprehensive way for general shape slots (with deep bars or double rotor cage) is a subject revisited in Chapter 9.
- Now with Chapters 5 and 6 in place, we have all basic parameters—magnetization inductance L_{lm} , leakage inductances L_{sl} , L_{rl}' and phase resistances R_s , R_r' .
- With rotor parameters reduced to the stator, we are ready to approach the basic equivalent circuit as a vehicle for performance computation.

6.15. REFERENCES

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